• Numerical solution of algebraic and transcendental equations:

1. Polynomial functions – linear, quadratic, cubic, quartic and n-order polynomials set as a summation series. Transcendental functions – $e^x$, $\ln x$ and $\sin x$ as infinite series.

2. Quadrants, fixed points, turning points and asymptotic properties of functions. Plotting $e^{\pm x}$, $\ln x$ and $x \ln x$. Plotting $\sin x$ on orbits and the linear axis. Plotting $\sin^2 x$. Plotting of the Gaussian function and the Lorentz function as a first-order approximation of the Gaussian function. Concept of scales.

3. Monomials – with positive exponents greater than unity and less. Plotting functions with odd and even exponents greater than unity. Fractional exponents – odd and even roots. Inflection points, coinciding roots and turning points. Monomials with negative exponents.


5. Matching of $e^x$ and $\cos x$ with Taylor polynomials of orders 1, 2 and 3 at $a = 0$. Matching of $\ln x$ and $\sin x$ with Taylor polynomials of orders 1, 2 and 3 at $a = 1$ and $a = 0$, respectively.

6. The theory of polynomials.

7. The basic principle and the application of the bisection method. The algorithm of bisection.

8. The basic principle of the Newton-Raphson method with examples.

9. The basic principle of the secant method with examples. Two methods of the initial guess values of the roots of the secant method.


11. Transforming a general cubic equation to the standard form of the cubic equation. The roots of the standard form of the cubic equation by Cardan’s method – real arithmetic roots along with the cube roots of unity. Solutions for $D > 0$, $D = 0$ and $D < 0$ (the irreducible case). Complex representation of solutions in the irreducible case. Vanishing discriminants and coinciding roots.

12. Factorising a quartic equation into a biquadratic. Solution of quartic equations by Ferrari’s method and by the Descartes method with examples.

• Interpolation:

1. Basic definitions, the Weirstrass approximation theorem and the Lagrange polynomial. Linear Lagrange interpolation of two points with examples. Quadratic polynomials to interpolate three points. Uniqueness of interpolating polynomials. Examples of quadratic and cubic Lagrange polynomials.

2. Divided differences, definition, the first-order divided difference, the mean-value theorem. Approximating a divided difference to the derivative of a function about the average position between points.

3. Second, third and higher-order divided differences. Properties of divided differences.

4. Newton’s divided-difference interpolation of linear, quadratic and cubic orders with examples.

5. Forward difference. Forward difference form of the interpolation polynomial.


• Differentiation and integration:
  
  1. Numerical integration – the trapezoidal rule. The trapezoidal formula for $n$ sub-intervals.
  2. Simpson’s rule and its derivation. Simpson’s rule for $n$ sub-intervals.
  5. Examples of numerical differentiation and error estimation in the central-difference formula.

• Numerical solution of systems of algebraic equations:
  
  2. Solvability of linear systems. Gaussian elimination, the augmented matrix.
  3. Gaussian elimination with general coefficients. A general $n$-order linear system.

• Numerical solution of differential equations:
  
  3. Euler’s method, the backward Euler method, the midpoint method and their truncation errors.
  4. Implicit methods and numerical stability.
  5. The trapezoidal method, Heun’s method and the Taylor method.

• Text and reference books:
  
  5. *An Introduction to Numerical Methods and Analysis*, J. F. Epperson, Wiley, New Jersey