Genesis and Objective

The course SC501: Essential Mathematics was introduced in the academic year 2008–09 as one of three core courses for all M.Tech. students. This was on the recommendations of the M.Tech. Review committee set up in the academic year 2007–08. Based on extensive discussions at that time, a syllabus was drawn up for the first round of the course. For details, see Appendix 1.

In the light of experience gained from the first round, the course content was moderated as laid out in Appendix 2.

In the third round, further changes were brought about, and the course content was organized in terms of themes, rather than as a list of topics as presented in text books (Real Analysis, Complex Analysis, Calculus, Group Theory, Linear Algebra, etc..) The following themes were chosen this time.

1. Spaces and Structures
2. Modeling and Measurement
3. (Algebraic) Preliminaries
4. Logic, Proof, and Rigour
5. Ideals and Approximations
6. Numbers and their Structures
7. Basics of Linear Algebra (Vector Spaces)
8. Network Analysis, Graph Theory, and Linear Algebra
9. Elements of (Finite) Group Theory
10. Posets, Lattices and Boolean Algebras
12. Metric Spaces, (Topological Spaces), Basics
13. Functional Analysis
14. Algebraic Varieties

The core idea for adopting this approach this time was to start off by asking the questions: The Course is titled, “Essential Mathematics”. Well, essential for whom, and for what? Why learn mathematics? Mathematics should be seen in what perspective? Mathematics as a language for expressing scientific thoughts? Mathematics as a hierarchy of conceptual metaphors? Mathematics
as a “tool” for modeling the real world? Mathematics as a never ceasing (never ending, ceaseless) exercise of conceptualizing abstractions upon abstractions?\footnote{Think of a goat. Which goat do you see? Not any particular one, you say, but rather a general one. In fact you are imagining an abstraction.}

My own deliberations on such questions have led me to believe that the objective of the course is perhaps best summed up by saying that it should help students acquire what may be called “a mathematical temper”.

It is not important in this connection which theorems (of linear algebra or functional analysis), and how many of them, students get to learn. What is important is that they begin to appreciate the line of enquiry underlying such theorems. In that sense, the instructor should have the freedom in the choice of specific mathematical concepts to be studied.

For the present round of the course, I have the following break up in mind.

\section*{A Tentative Table of Contents}

Based on the experience of the last three rounds, I have right now in mind a breakup of the lectures on the following lines.

<table>
<thead>
<tr>
<th>TOPICS</th>
<th>HRS($\leq$)</th>
<th>HRS(Planned)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Structures</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Linear Algebra</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Measurement Theory and Modeling</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Equivalence and Order</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Writing, Journalizing, and Scholarship</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Symmetries and Group Theory</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Functional Analysis</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Probabilistic Inference</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Questions and Answers</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td><strong>ALL TOGETHER</strong></td>
<td><strong>47</strong></td>
<td><strong>40</strong></td>
</tr>
</tbody>
</table>

\section*{Outcome}

At the conclusion of this course, students should have a sound understanding of what mathematics is about, and should have acquired a level of mathematical literacy that would enable them to see its relevance in their own domain of knowledge.
Appendices

A Background

SC501: Essential Mathematics - 3-0-0-3
Instructor(s): Prof Samaresh Chatterji and Prof V P Sinha

Background: During the 2007-08 academic year, a formal committee (M Tech Curriculum Review Committee - MTCRC) was set up to review the curriculum of the M Tech program: this was the first formal review of the program, though minor changes in the curriculum had been made from time to time. The outcome of the review was a considerable change in the curriculum. The most noteworthy change was the introduction of formal streams in the program; for the academic year 2008-09, four such streams were introduced: Communication Systems, Computer Networks, Machine Intelligence, and VLSI and Embedded Systems. The second major change was the introduction of three core courses to be undertaken by all the students in the first semester: Essential Mathematics, Probability Theory, and Algorithms. The course SC501 came into the curriculum as a result, and was offered for the first time during the Autumn Semester 2008-09.

Contents: The contents for the course evolved over a period of time involving discussions with members of the MTCRC, other faculty members, and earlier students of the M Tech and Ph D programs. During this period, several possible textbooks were examined and the approach of "All the Mathematics You Missed" by Thomas A Garrity was found to be very promising. It was therefore agreed to use this book as a textbook. However, it was clear that not all the chapters could be covered, and so the following chapters which were deemed to be of most significance for engineering students were eventually selected for coverage: Chapter 1: Linear Algebra; Chapter 2: Real Analysis; Chapter 4: Point Set Topology; Chapter 6: Complex Analysis; Chapter 13: Fourier Analysis; Chapter 14: Differential Equations. Most of the course was taught by Prof Chatterji; Prof Sinha taught the chapter on Fourier Analysis. Additional details are captured in the course outline appended in the course file.

Review: As the course progressed, it became evident that the original course was overly ambitious in scope, and considerable adjustments had to be made in terms of content. It was realized that the textbook would have to be supplemented by additional material. But the relatively terse coverage in the book had many gaps which the students could not individually cover given their mathematical background. More seriously, the students were not equipped to deal with a conceptual approach to mathematics. They expected a bunch of techniques to apply, rather than concepts which would help them to understand and connect seemingly disparate applications. The assignments were usually tackled by finding the solutions in some textbook, rather than used as an opportunity to enlarge their own understanding. Consequently, the performance of the students on the examinations was almost uniformly very poor.

For the Future: A post-course interaction with the students indicated that most of them felt that a mathematics course was desirable, though perhaps different courses should be worked out for the different specialization streams. The topics which all felt were needed were Linear Algebra and Real Analysis.

B Course Outline

SC501: Essential Mathematics - 3-0-0-3
Instructor: Prof Samaresh Chatterji

Offered to MTech students as a core course.
Objective: MTech students frequently find that their mathematical foundation is inadequate to pursue research for their thesis, and it is difficult for them to achieve the required level entirely through self-study. This course has the objective of providing the essential knowledge required to remove this inadequacy. The approach we are going to adopt is to take as the starting point selected chapters from the textbook, supplementing the same with additional material and problem sets. From the pedagogical point of view, the emphasis will be on mathematical rigor, working with mathematical concepts, and problem solving skills.

Contents: Chapter 1: Linear Algebra: Rn; vector spaces; bases and dimension; linear transformations and matrices; invertibility; similarity; eigenvalues, eigenvectors, diagonalization Chapter 2: Real Analysis: and formulations; limits; continuity; differentiation, integraton and the fundamental theorem of calculus; convergence of functions; nowhere differentiable functions Chapter 4: Point Set Topology: basic concepts; metric spaces; compactness and connectedness Chapter 6: Complex Analysis: analytic functions; Cauchy-Riemann equations; complex integration; power series; conformal mapping and the Riemann Mapping Theorem Chapter 13: Fourier Analysis: finite discrete sequences via Fourier transforms Chapter 14: Differential Equations: ODEs: Existence and Uniqueness Conditions; PDEs: the Laplacian; the Heat Equation; the Wave Equation; Integrability Conditions

Outcome: Students should be able to read research papers, understand the commonly used mathematical concepts and terms appearing in them, and be familiar with the fundamental results involving them.


Evaluation Methodology: Assignments 20-2525-30assistance of the TA for their difficulties and discussing the assignment solutions

Instructor: V.P. Sinha
July 19, 2012