1. A queue is implemented using an array of size $N$. Variable $f$ is equal to the index of the cell containing the next element to be deleted, and variable $r$ is equal to the index of the cell where the next element will be added. Give a single formula to calculate the number of elements in the queue, if the queue is circular.

2. Write a traversal algorithm for proper binary trees in which the nodes at depth $d$ are visited before the nodes at depth $d+1$.

3. Let us define a function $p$ from the set of nodes of a proper binary tree $T$ to the set of natural numbers as follows:

   - If $v$ is the root of $T$, then $p(v) = 1$
   - If $v$ is the left child of node $u$, then $p(v) = 2 \cdot p(u)$.
   - If $v$ is the right child of node $u$, then $p(v) = 2 \cdot p(u) + 1$.

   If $n$ is the total number of nodes, then show that the maximum value of $p(v)$ satisfies $n \leq p(v) \leq 2^{(n+1)/2} - 1$

4. Show that the summation $\sum_{i=1}^{n} \lceil \log_2(n/i) \rceil$ is $O(n)$.

5. Given two ordered sequences corresponding to sets $A$ and $B$, write an algorithm to compute a sequence corresponding to the set (a) $A \cup B$ (b) $A \cap B$

6. Let $A$ and $B$ be two sequences of $n$ integers each. Given an integer $x$,
describe an $n^2$-time algorithm for determining if there is an integer $a$ in $A$ and an integer $b$ in $B$ such that $x = a + b$. Can you come up with an $O(n \log n)$-time algorithm for the same problem.

7. What is the probability that in a random ordering of bridge card deck, the ace of spade precedes the king of hearts.

8. Prove that if two events $A$ and $B$ are independent, then their respective complements are also independent.

9. Let $C$ be the probability space of all the n-term sequences of 0s and 1s. Define a random variable $f$ s.t $\forall s \in C : f(s) =$ number of 1s in $s$. Determine the expected value of $f$ in multiple ways.

10. Let there be $n$ hunters and $n$ rabbits. Each of the $n$ hunters selects a rabbit at random and then all the hunters shoot at once. Find the expected number of surviving rabbits.