Artificial Intelligence
Applying Utility Theory for Expectimax

Lecture 5
Two important questions ....

- How do we convince ourselves that Utility Function is good for our agent?

- How do we get the Utility Values?
How do we know that Utility is good for agent?
Utility Theory

- Principle of maximum expected utility:
  - A rational agent should choose the action which maximizes its expected utility, given its knowledge

- Where do utilities come from?
- How do we know such utilities even exist?
- Why are we taking expectations of utilities (not, e.g., minimax)?
- What if our behavior can’t be described by utilities?
Utilities: Dealing with uncertainty

Which is better?
By what factor?
“Preferences”

- An agent chooses among:
  - Prizes: $A$, $B$, etc.
  - Lotteries: situations with uncertain prizes

$L = [p, A; (1-p), B]$

- Notation:

Can mean that agent likes B or A or just doesn’t care!

Does NOT mean:

- $A > B$
- $A \sim B$
- $A \succeq B$

Why??

$P > (1-p)$

$A$ is equivalent to $B$ in terms of choice

Weaker condition

$A$ preferred over $B$

indifference between $A$ and $B$

$B$ not preferred over $A$
“Rational” Preferences

- Preferences of a rational agent must obey constraints.
  - The axioms of rationality:
    - Orderability
      \[(A \succ B) \lor (B \succ A) \lor (A \sim B)\]
    - Transitivity
      \[(A \succ B) \land (B \succ C) \Rightarrow (A \succ C)\]
    - Continuity
      \[A \succ B \succ C \Rightarrow \exists p \ [p, A; \ 1 - p, C] \sim B\]
    - Substitutability
      \[A \sim B \Rightarrow [p, A; \ 1 - p, C] \sim [p, B; \ 1 - p, C]\]
    - Monotonicity
      \[A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; \ 1 - p, B] \succeq [q, A; \ 1 - q, B])\]

What if agent violates this?

Probabilistic Trade-off
between two extremes (A & C)

What is the difference?

This exists

Theorem: Rational preferences imply behavior describable as maximization of expected utility
MEU Principle

- **Theorem:**
  - [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function $U$ such that:

  $$U(A) \geq U(B) \iff A \succeq B$$

  $$U([p_1, S_1; \ldots ; p_n, S_n]) = \sum_i p_i U(S_i)$$

- **Maximum expected utility (MEU) principle:**
  - Choose the action that maximizes expected utility
  - **Note:** an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe, reflex vacuum cleaner
Moral of the story: We know Utility is good!
Utility Scaling: Different Approaches

- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$

- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.

- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk

- Note: behavior is invariant under positive linear transformation

$$ U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0 $$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes
How do we get the Utility Values?
Human Utility Function – how would a human think?

- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment of human utilities:
  - Compare a state $A$ to a standard lottery $L_p$ between
    - “best possible prize” $u_+$ with probability $p$
    - “worst possible catastrophe” $u_-$ with probability $1-p$
  - Adjust lottery probability $p$ until $A \sim L_p$
  - Resulting $p$ is a utility in $[0,1]$

Would you want to walk away with this or play the game? Ans: I am confused!!

No more affects $A$;
Whatever is the outcome of the lottery, the preference of $A$ no more changes

pay $30$
A pictorial representation ...

Hee Hee Haa Haa!!!!!
Either play (and die!) or pay me money

Should I play or pay?? -> How much do I care (i.e. value) for my money
Money Utility Model

- Money does not behave as a utility function, but we can talk about the utility of having money (or being in debt).
- Given a lottery $L = [p, X; (1-p), Y]$:
  - The expected monetary value $EMV(L)$ is $pX + (1-p)Y$
  - $U(L) = pU(X) + (1-p)U(Y)$

**Typically, $U(L) < U(EMV(L))$: why?**
- In this sense, people are risk-averse.
- When deep in debt, we are risk-prone.

- Utility curve: for what probability $p$ am I indifferent between:
  - Some sure outcome $x$
  - A lottery $[p,M; (1-p),0]$, $M$ large.
“Insurance”

- Consider the lottery $[0.5, 1000; 0.5, 0]$
  - What is its expected monetary value? ($500$)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - $400 for most people
  - Difference of $100 is the insurance premium
    - There’s an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!
A pictorial representation
Are human’s rational?

- Famous example of Allais (1953)
  - A: [0.8,$4k; 0.2,$0]
  - B: [1.0,$3k; 0.0,$0]
  - C: [0.2,$4k; 0.8,$0]
  - D: [0.25,$3k; 0.75,$0]
Result of the experiment

- Most people prefer $B > A$, $C > D$
- But if $U(\$0) = 0$, then
  - $B > A \Rightarrow U(\$3k) > 0.8 \ U(\$4k)$
  - $C > D \Rightarrow 0.8 \ U(\$4k) > U(\$3k)$

Scaled Up .. Which should not matter for a proper Utility function

Paradox?