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Mutual Information and Energy Tradeoff in Correlated Wireless Sensor Networks

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***Abstract*— We consider comparison of Mutual Information (MI)-Energy tradeoff in correlated wireless sensor network (WSN) for cooperative multiple input and multiple output (MIMO) and non-cooperative (referred to as single input and single output (SISO)) transmission schemes. Our numerical results indicate that for short distances SISO schemes achieve better MI-Energy tradeoff than the cooperative MIMO schemes, while for long distances the cooperative MIMO schemes achieve better MI-Energy tradeoff. We next consider the issue of sensor placement in a correlated WSN. Specifically, we prove that the spacing between nodes to maximize MI is in general an increasing function of observation SNR. From this structural result, we conclude that at high SNR, the various stages of the WSN can be separated by relatively large distances than at low SNR.**

I. INTRODUCTION

A wireless sensor network (WSN) consists of large number of spatially distributed devices called *nodes* which cooperate to accomplish various high level tasks. Typically the nodes are equipped with small batteries, and hence are subject to severe energy constraints. Cooperative transmission based techniques have been considered to reduce energy consumption of the nodes, and hence prolong the life time of the network [1]. Specifically, the authors in [1] assume that the sensor nodes exchange information and transmit cooperatively, using the well known multiple input and multiple output (MIMO) encoding technique due to Alamouti. The cooperative MIMO transmissions are energy efficient compared to the non-cooperative tranmission (referred to as single input and single output (SISO) transmissions), especially for long distances [1].

Another unique feature of a WSN is the high degree of *spatial correlation* in the data sensed by the nodes. The spatial correlation decreases with an increase in the distance between nodes of the WSN. The work in [2] considers distributed source coding (DSC) to exploit the correlation in WSNs. While the works in [3] and [4] consider route selection in correlated WSNs.

This paper has two main contributions with respect to the correlated WSNs. First, we study the tradeoff between Mutual Information (MI) and energy consumption in correlated WSNs employing SISO and MIMO data gathering techniques. It is well known that the traditional MIMO transmissions achieve better MI and energy tradeoff than the SISO transmissions. However, in a WSN employing cooperative MIMO transmission scheme, it is not clear if MIMO transmissions achieve better MI and energy tradeoff compared to the SISO transmission scheme, because of the additional overhead in terms of local communication.

The second important contribution of this paper is the impact of observation SNR on sensor placement so that the mutual information is maximized.Further, we also study the effect of observation SNR on sensor spacing so that the error achieved at the destination is minimized. To accomplish this, we use concepts in *Monotone Comparative Statics (MCS)* [5]. Monotone Comparative Statics are used extensively in economics literature to study as how the optimal solution of a parametrized optimization problem varies monotonically with respect to the parameter [5]. The main results of this paper are summarized as follows:

A. Main Results

1) We model the correlation in WSNs according to a linear Gauss-Markov model, and study the MI-Energy tradeoff for SISO and MIMO data gathering schemes. From our numerical results, we observe that MIMO achieves better MI-Energy tradeoff than the SISO transmissions for long distances. However for small distance ranges SISO achieves better MI-Energy tradeoff.
2) Using the results in MCS, we prove the spacing between nodes to maximize MI in a correlated WSN, to be an increasing function of the observation SNR.
3) Using this structural result, and Mean Square Error (MSE) as a performance measure, we conclude that at high observation SNR, correlation does not have significant impact on performance. However at low observation SNR, we conclude that the performance depends critically on the correlation.

II. WIRELESS SENSOR NETWORK (WSN) MODEL AND ASSUMPTIONS

We consider data gathering using WSN. The nodes in the WSN have a fixed transmission range, and hence employ multi-hop routing. We assume there are $(I+1)$ stages in the WSN, where the first I stages denote the data gathering stages, while the $(I+1)^{\text{th}}$ stage denotes the sink. Each of the I data-gathering stages may contain one (or) more than one node depending on whether it is a SISO (or) Cooperative MIMO data gathering scheme.

In the SISO case, the only node present in a given stage senses, quantizes and transmits to the next stage. In the MIMO case a node called the *sensing node* makes an observation of the underlying process, quantizes, and broadcasts locally to other nodes in that stage for cooperative transmission based on space-time block codes (STBC) [6]. However, in this paper we restrict ourselves to the case of two nodes per stage, and the STBC in this case is commonly referred to as the *Alamouti code*.

A. Assumptions

Owing to the close proximity of the various stages of the network, the data sensed by the nodes in the various stages of the network can be assumed *spatially correlated*. For the purpose of our analysis, we assume the signal along the route to the sink forms a one dimensional stationary Gauss-Markov process as in [3] and [4]. The Gauss-Markov assumption on the underlying signal process leads us to the following state-space representation of the signal process:

$$\begin{aligned} x_k &= a_{k-1}x_{k-1} + u_{k-1} \text{ ,for } k = 1, 2, \cdots, I, \\ u_{k-1} &\sim \mathcal{N}\left(0, P_0(1 - a_{k-1}^2)\right) , \end{aligned} \tag{1}$$

where a_{k-1} denotes the correlation between $(k-1)^{\text{th}}$ and k^{th} stages of the network.

In general the models used for correlation are classified into four standard groups: *Spherical, Power Exponential, Rational Quadratic* and *Matérn* [7]. The key properties such as non-negativity of correlation, and decrease in correlation with distance are satisfied by all the four models. However for our analysis in this paper, we model correlation a_{k-1} using the popular *Power Exponential* model as $a_{k-1} = e^{-Ad_{k-1}}$, where A is the field diffusion coefficient and d_{k-1} denotes the distance between $(k-1)^{\text{th}}$ and k^{th} stage of the network.

Further each stage of the network on the way to the destination observes a noisy version of the process i.e.,

$$y_k = x_k + n_k \ , \ n_k \sim \mathcal{N}\left(0, \sigma^2\right) , \tag{2}$$

where σ^2 denotes the variance of observation noise.

Due to stationarity of the random field, the observation SNR at each stage of the network is same and is defined as

$$\Gamma \triangleq \frac{P_0}{\sigma^2} . \tag{3}$$

In a WSN, the processing energy of each node can be significant. Hence, the total energy consumption of each node includes transmission and circuit energy consumption as in [1] [8]. We model the communication channel according to a Rayleigh fading model. We assume each sensor node employs 4-ary quadrature amplitude modulation (M-QAM) (i.e., transmits 2 bits per modulation symbol) for digital modulation. We assume that the sensors quantize the observation with high precision, and hence ignore the quantization error.

III. Formulation of MI-Energy Tradeoff Optimization Problem in a Correlated WSN

A. Total Energy Consumption with SISO Data Gathering

In SISO transmission since there is no local information circulation, the single node present in a given stage senses, quantizes, modulates, and transmits to the next stage.

The total power consumed in a sensor node employing 4-QAM while transmitting from stage $(k-1)$ to (k) is given as [1]

$$P_{SISO}^{k-1} = \underbrace{\frac{1}{2}(1+\alpha)\left(\frac{1}{p_b}\right)N_0G_0d_{k-1}^2B}_{\text{Transmission power}} + \underbrace{(P_T + P_R)}_{\text{Circuit power}} , \tag{4}$$

where α is a constant defined by the power amplifier efficiency, p_b is the desired probability of error, N_0 is the noise spectral density, G_0 is the attenuation factor, d_{k-1} denotes the distance between $(k-1)^{\text{th}}$ and k^{th} stage of the network, B is the symbol rate, and P_T, P_R denote the transmitter and receiver circuit power consumption.

Thus the energy consumption per bit in transmitting from stages $(k-1)$ to k using SISO transmission can be written as

$$\begin{aligned} E_{SISO}^{k-1} &= \left(\frac{P_{SISO}^{k-1}}{B}\right)\frac{1}{2} , \\ &= \frac{1}{4}(1+\alpha)\left(\frac{1}{p_b}\right)N_0G_0d_{k-1}^2 + \frac{(P_T+P_R)}{B}\frac{1}{2} , \end{aligned} \tag{5}$$

the factor of 2 in the above equation accounts for the fact that two bits are transmitted in a modulation symbol.

B. Total Energy Consumption with MIMO Data Gathering

As mentioned in Section II, for cooperative transmission to be possible there needs to be local information circulation. To accomplish this, we assume a node in each stage of the network makes an observation, quantizes, modulates and and broadcasts to the other node in that stage for cooperative transmission.

The energy consumption for this local broadcast in k^{th} stage can be written as in Equation (5) to be

$$E_{local} = \frac{1}{4}(1+\alpha)\left(\frac{1}{p_b}\right)G_0N_0d_{\max}^2 + 2.\frac{(P_T+P_R)}{2B\ b_{local}} , \tag{6}$$

where $d_{\max}$ denotes the cluster radius of stage. The factor of 2 in the second term of Equation (6) is due to the fact that during local broadcast the circuits of the receiving node are also turned on.

The total power consumed for cooperative long haul transmission from the $(k-1)^{\text{th}}$ stage to the k^{th} stage can be written as

$$P_{LH}^{k-1} = \underbrace{2\sqrt{2}(1+\alpha)\left(\frac{4}{p_b}\right)^{\frac{1}{2}}G_0N_0d_{k-1}^2B}_{\text{Transmission Power}} + \underbrace{2(P_T+P_R)}_{\text{Circuit Power}} , \tag{7}$$

where, as before, α is a constant defined by the power amplifier efficiency, G_0 is the attenuation factor, d_{k-1} denotes the distance between $(k-1)^{\text{th}}$ and k^{th} stages respectively, and B denotes the symbol rate and P_T, P_R denotes the power consumption in transmitter and receiver circuits.

Thus, the total energy consumption per bit for transmission from $(k-1)^{\text{th}}$ stage to the k^{th} stage can be approximately written as [1]

$$E_{MIMO}^{k-1} = \left(\frac{P_{LH}^{k-1}}{B}\right)\frac{1}{2} + E_{local}\ , \tag{8}$$

where E_{local} is given by Equation (6).

C. Computation of MI between the Signal and Observation Process

The following theorem computes the Mutual Information (MI) between the signal and observation process of data gathering stages $2, 3, \cdots, I$ for the model assumed in Equation (1) and Equation (2).

Theorem 1: *Define* $X \triangleq (x_2, x_3, \cdots, x_I)$ *and* $Y \triangleq (y_2, y_3, \cdots, y_I)$ *where* x_i *and* y_i *denote the instance of signal and observation process respectively, then the mutual information* $I(X, Y)$ *between the signal sequence* (X) *and the observation sequence* (Y) *for the linear Gauss-Markov model in equation (1) and Equation (2) is given by*

$$I(X;Y) = \frac{1}{2}\sum_{k=2}^{I}\log_2\left(1 + \frac{\Sigma_{k|k-1}}{\sigma^2}\right)\ , \tag{9}$$

Proof: The theorem follows straightforwardly from the basic definition of MI, and noting that the probability density function of $Y|X$ is independent multi- variate Gaussian. ■ Using the Kalman recursion [9], we can write the expression for predictor covariance $\Sigma_{k|k-1}$ as

$$\frac{\Sigma_{k|k-1}}{\sigma^2} = a_{k-1}^2\gamma_{k-1} + \Gamma\left(1 - a_{k-1}^2\right)\ , \tag{10}$$

$$\text{where } \gamma_{k-1} = \frac{\Sigma_{k-1|k-2}}{\Sigma_{k-1|k-2} + \sigma^2}. \tag{11}$$

Using Equation (10), we can express RHS of Equation (9) as

$$I(X;Y) = \frac{1}{2}\sum_{k=2}^{I}\log_2\left\{1 + \Gamma - (\Gamma - \gamma_{k-1})\, a_{k-1}^2\right\}\ ,$$
$$\overset{\text{high SNR}}{\approx} \frac{1}{2}\sum_{k=2}^{I}\log_2\left\{1 + \Gamma - (\Gamma - 1)\, a_{k-1}^2\right\}\ , \tag{12}$$

where Γ denotes the observation SNR, γ_{k-1} is defined as in Equation (11) and a_{k-1} denotes the correlation between $(k-1)^{\text{th}}$ and k^{th} stage respectively.

With the Power exponential model assumed to model the correlation i.e., $a_{k-1} = e^{-Ad_{k-1}}$, we can express Equation (12) in terms of interstage spacing $d_1, d_2, \cdots, d_{I-1}$ as

$$I(d_1, d_2, \cdots, d_{I-1}; \Gamma) = \frac{1}{2}\sum_{k=2}^{I}\log_2\{1 + \Gamma - (\Gamma - 1)\, e^{-2Ad_{k-1}}\}\ , \tag{13}$$

where d_{k-1} is the distance between $(k-1)^{\text{th}}$ and k^{th} stages of the network.

D. MI-Energy Tradeoff in a Correlated WSN

To study the tradeoff between MI and total energy consumption in the network for MIMO and SISO data gathering schemes, we formulate the optimization problem as follows:

$$\underset{[d_1, d_2, \cdots, d_{I-1}]}{\text{Minimize}}\ \lambda\sum_{k=2}^{I} E_{SISO}(d_{k-1}) - I(d_1, d_2, \cdots, d_{I-1}; \Gamma)$$

(AND)

$$\underset{[d_1, d_2, \cdots, d_{I-1}]}{\text{Minimize}}\ \lambda\sum_{k=2}^{I} E_{MIMO}(d_{k-1}) - I(d_1, d_2, \cdots, d_{I-1}; \Gamma)$$

$$\text{Subject to: } 0 \leq d_{k-1} \leq D \text{ for } k = 2, 3, \cdots, I \tag{14}$$

where $I(.)$ is given by Equation (13), E_{SISO} and E_{MIMO} are energy consumption per bit values for SISO and MIMO data gathering schemes given by equations (5) and (8) respectively, $\lambda\ (> 0)$ is the scanning parameter, and D is the maximum distance range that the sensors can transmit, and is typically determined by the peak power of the sensor nodes.

By varying the scanning parameter λ, we obtain the tradeoff curve between total energy consumption and MI. For a given λ, the optimization problem in Equation (14) is convex [1] and hence can be solved efficiently [10].

To give a numerical example we consider a ten stage network i.e., $(I+1) = 10$, wherein stages $1, 2, \cdots, 9$ correspond to data gathering stages and the 10^{th} stage corresponds to the sink. We choose the system parameter values as in [1]. We set the probability of error $(p_b) = 10^{-6}$. Our optimization variables in this case are $d_1, d_2, \cdots, d_8$, which correspond to the distance between successive data gathering stages of the network. We set the maximum distance (D) that the nodes can transmit to be 30 metres. Finally, we set the observation SNR $\Gamma = 10$ dB, and the field diffusion coefficient $(A) = 0.1$.

The optimal energy-MI tradeoff values and the optimal distance values for MIMO and SISO data gathering schemes are shown in Table I. *From the values in the table it can be concluded that for small distance ranges i.e., upto 6.5 metres, SISO data gathering scheme requires less energy compared to the MIMO data gathering scheme to achieve the same amount of MI. But for large distances (roughly greater than 7.5 metres) MIMO data gathering scheme achieves better MI and energy tradeoff.*

Table I also shows the value of Mean Square Error (MSE) achieved at the destination for the corresponding optimal distance values. The MSE values are obtained by using the well-known Kalman filter recursion. *From Table I it can be observed that in general as the distance increases, the MSE increases. This is to be expected since an increase in the distance reduces the correlation in the data sensed by nodes, which leads to performance degradation. Further it can be observed that for small distances, SISO transmission consumes*

[1] Since the energy consumption is quadratic in d_{k-1} and the negative of MI can be verified to be convex.

less energy than the MIMO data gathering scheme. However for large distances, cooperative MIMO based transmission scheme achieves the same MSE at a lower energy compared to the SISO data gathering scheme.

Thus from this simple numerical example we can conclude that for small distances SISO based data gathering achieves better energy-MI (or MSE) tradeoff than the MIMO based scheme. However for moderate and large distances MIMO based scheme achieves better energy-MI (or MSE) tradeoff.

IV. Effect of Observation SNR on Optimal Sensor Spacing

A. Structural Result

In this subsection we study the impact of observation SNR (Γ) on sensor placement so that the mutual information is maximized.

To put the problem into perspective consider the following optimization problem, wherein we consider optimal placement of sensors to maximize mutual information:

$$\begin{aligned} &\underset{[d_1 d_2 \cdots, d_{I-1}]}{\text{Maximize}} \; I([d_1, d_2, \cdots, d_{I-1}]; \Gamma) \\ &\text{s.t.: } 0 \leq d_{k-1} \leq D \text{ for } k = 2, 3, \cdots, I. \end{aligned} \tag{15}$$

where $I(.)$ is the MI from source to destination given by Equation (13). Before we proceed with our structural result, we state the following simplified version of Topkis's Monotonicity theorem in [5]. This simplified version provided in [11] is adequate to prove the structural result in our paper.

Theorem 2: *For the optimization problem below:*

$$\textit{maximize } F(s, a) \textit{ subject to } s \in S$$

Suppose that S forms a lattice and has at least one solution for each $a \in A$. Suppose also that F is supermodular, then the optimal s is always increasing in a.

The following remark provides a simple condition to verify if a function is supermodular.

Remark 1: If the function F is continuously differentiable then and if $\frac{\partial^2 F(s,a)}{\partial a \partial s} (>) \geq 0$, then the function F is said to be supermodular [11].

We now state the structural result on the variation of optimal spacing d_{k-1} w.r.t the observation SNR (Γ) as the following theorem.

Theorem 3: *The optimal spacing $[d_1, d_2, \cdots, d_{I-1}]^*$ between sensors to maximize the MI from source to destination is in general an increasing function of observation SNR Γ. i.e.*

$$\begin{aligned} &[d_1, d_2, \cdots, d_{I-1}]^* \triangleq \arg \max_{[d_1, d_2, \cdots, d_{I-1}]} I\,(.;.) \\ &\textit{s.t. } 0 \leq d_{k-1} \leq D \textit{ for } k = 2, 3, \cdots, I \end{aligned} \tag{16}$$

is an increasing function of Γ, where $I(.;.)$ is given by Equation (13)

Proof: The proof of the above theorem follows by noting that the constraints form a lattice, the function $I(.;.)$ in Equation (13) is supermodular, and invoking the Theorem 2. The function I in Equation (13) can be verified to be supermodular, by noting that every term in Equation (13) is supermodular, and sum of supermodular functions is indeed supermodular. ■

Interpretation of Theorem 3: At high observation SNR since the signal component is strong, hence correlation in data does not contribute additional information, hence in this case the sensor nodes can be separated by large distance. However at low observation SNR, since the signal is weak, the correlation in data contributes additional information, hence in this case the sensor nodes need to be placed closely so that the correlation in data can be exploited.

B. Discussion

The increase in optimal spacing with observation SNR implies correlation does not have much impact on performance at high observation SNR, since correlation decreases with increase in distance. While at low observation SNR correlation can affect the performance significantly, since at low observation SNR sensors need to be more closer than at high observation SNR as proved in Theorem 3.

To verify this claim we present a simulation case study with Mean Square Error (MSE) as a performance measure at high and low observation SNR. For the high SNR case we assume observation SNR $(\Gamma) = 30$ dB, while for the low SNR case we set the observation SNR $(\Gamma) = -10$ dB. For both cases we set the distance between nodes as equal to $1, 5$ and 20 metres respectively. We set the value of field diffusion coefficient (A) equal to 0.1. For the network model we assume the same ten stage network.

Figure 1 shows the plot of MSE versus stage index of the network for high observation SNR. Here we assume $\Gamma = 30$ dB. The figure shows that the MSE achieved when node spacing is equal to $1, 5$ and 20 metres is almost same. This is because at high observation SNR, the signal component in the observation is strong and has greater influence on MSE than the noise averaging effect of correlation. Consequently, the performance is independent of correlation and hence spacing between nodes can be large when observation SNR is high (since correlation does not convey any new information), as proved in Theorem 3.

Figure 2 shows the plot of MSE versus stage index of the network for low observation SNR. Here we assume $\Gamma = -10$ dB. The figure shows that node spacing corresponding to 1 metre leads to lower value of MSE compared to the case when node spacing is equal to 5 and 10 metres respectively. This is because at low SNR, the signal component in the observation is weak and hence the MSE achieved at the destination depends significantly on the noise averaging effect of the correlation. Thus, at low observation SNR, performance depends on correlation, and hence spacing between nodes has to be small at low observation SNR, to exploit the correlation as proved in Theorem 3.

This analysis provides valuable insight into the placement of sensors for a given application. For example if the observation SNR is high, then the sensors can afford greater spacing between them without loss in performance. On the other hand when the observation SNR is low, the sensors need to be placed closely to exploit the correlation.

V. Conclusions

We considered the issue of MI-Energy tradeoff in correlated wireless sensor networks for two representative schemes namely, the cooperative MIMO transmission scheme, and the non-cooperative SISO transmission scheme. From our numerical results, it can be concluded that for short distances the SISO scheme achieves better MI-Energy tradeoff than the cooperative MIMO scheme. However for long distances, the MIMO scheme achieves better MI-Energy tradeoff. We also prove an important structural result on the variation of optimal spacing with respect to the observation SNR. Specifically, we showed that the optimal spacing between the successive stages of the network is in general an increasing function of the observation SNR. From this structural result, we concluded that at high SNR, the various stages of the WSN can be separated by relatively large distances, than at low SNR.

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TABLE I
MI-ENERGY TRADEOFF FOR MIMO AND SISO DATA GATHERING SCHEMES

MI in bits	Optimal Spacing (d^*) in m	MSE	E_{SISO} in Joules	E_{MIMO} in Joules
11.90	3.20	0.0756	6.7730×10^{-5}	1.5239×10^{-4}
14.00	6.70	0.0874	1.2670×10^{-4}	1.5249×10^{-4}
14.35	7.85	0.0876	1.5486×10^{-4}	1.5254×10^{-4}
14.77	9.85	0.0883	2.1509×10^{-4}	1.5264×10^{-4}
14.94	10.96	0.0889	2.5466×10^{-4}	1.5271×10^{-4}
15.07	12.10	0.0893	2.9904×10^{-4}	1.5278×10^{-4}
15.31	15.30	0.0901	4.4759×10^{-4}	1.5304×10^{-4}
15.55	30	0.0909	16×10^{-4}	1.5496×10^{-4}

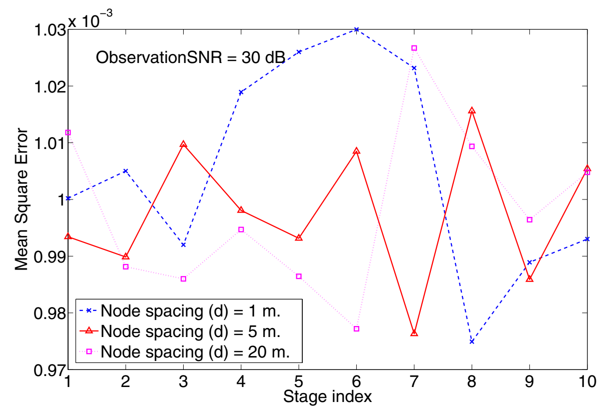

Fig. 1. The figure shows the plot of mean square error versus the stage index. We fix the observation SNR (Γ) equal to 30 dB and assume the node spacing to be 1 m, 5 m and 20 m respectively.

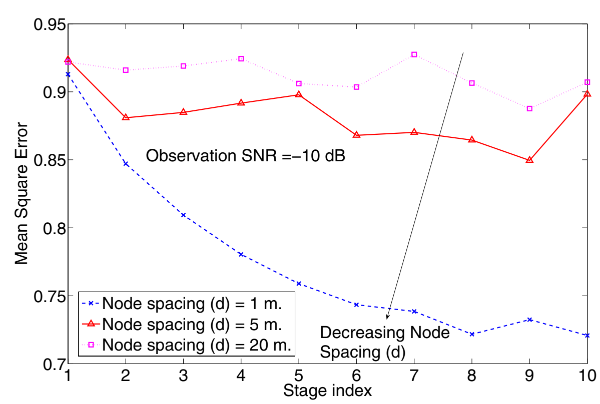

Fig. 2. The figure shows the plot of mean square error versus the stage index. We fix the observation SNR (Γ) equal to -10 dB and assume the node spacing to be 1 m, 5 m and 20 m respectively.