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JOINT RATE AND CLUSTER OPTIMIZATION IN COOPERATIVE MIMO SENSOR NETWORKS

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ABSTRACT

We design transmission schemes in cooperative MIMO sensor networks that optimize the choice of rate and cluster for minimizing the energy consumption subject to a given average probability of error. In general it was observed that, the number of clusters to optimize the energy consumption is a non-increasing function of long haul distance. By virtue of this, we conclude that as the long haul distance increases the cooperation among the nodes increases. Also, for large long haul distances the optimal number of clusters is shown to converge to a constant. Next, we propose a provably convergent block coordinate descent algorithm to determine the optimal joint rate and number of clusters. Through our numerical results it was observed that a cluster optimized cooperative MIMO transmission scheme can be more energy efficient than a rate only optimized scheme. Also a joint rate and cluster optimized transmission scheme can yield large scale energy savings for short and medium range applications compared to the rate only or cluster only optimized transmission schemes.

1. INTRODUCTION

Cooperation based multiple input and multiple output (MIMO) communication architectures have been recently proposed to improve the energy efficiency of sensor networks [1]. In [1], it had been shown that employing an Alamouti code based information transmission scheme can lead to large scale energy savings in the long haul applications. This paper also disproves the popular belief that MIMO based transmission schemes are always energy efficient, especially when the total energy (circuit energy plus transmission energy) is taken into account.

Another application specific protocol architecture called low-energy adaptive clustering hierarchy (LEACH) had been proposed for wireless microsensor networks in [2]. In LEACH, all the nodes would transmit the data to a clustering head (CH), which in turn would relay the data to the base station (BS). Acting as a CH could be quiet energy intensive, hence LEACH employs randomized rotation of CH.

In this work, assuming random distribution of the sensor nodes, we analyze the energy efficiency of cluster based cooperative MIMO techniques. To start with, we obtain an expression for the total energy consumption in terms of the two parameters that need to be optimized namely, number of clusters and rate. Using this expression, we analyze as how the optimal number of clusters vary with the long haul distance. We also compare the energy consumption of the three approaches namely: rate optimization, cluster optimization and joint rate and cluster optimization.

The rest of this paper is organized as follows: In Section 2, we present system model description and assumptions. In Section 3, we derive the energy consumption in a cluster based cooperative MIMO sensor network. Section 4 presents optimization methods to determine the optimal number of clusters and the optimal joint rate and number of clusters. Section 5 contains discussion on implementation aspects of the proposed cluster based cooperative MIMO scheme. Finally, Section 6 contains some important conclusions of this work.

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2. SYSTEM MODEL

2.1. Cooperative MIMO

We consider the scenario where a group of homogeneous sensors are dropped to keep track of an event of interest. These nodes are divided into small groups called *clusters*. The nodes with in each cluster, collaborate to form a virtual multiple antenna system and transmit information to a data gathering node (for example, a base station) using *orthogonal space time block codes (STBC)* [3]. In our analysis, we consider the energy consumption for transmission and in processing circuits. To simplify our analysis, we neglect the energy consumption for base band processing such as source coding, channel coding and digital modulation. Further, we assume the nodes are capable of operating in multiple modes namely: active, transient and sleep. Operating the nodes on a multi-mode basis could lead to significant energy savings [4].

3. ENERGY CONSUMPTION OF CLUSTER BASED COOPERATIVE MIMO TECHNIQUES

In this section, we present the total energy consumption of the cluster based cooperative MIMO techniques.

For our analysis, we assume N sensors are dropped uniformly in a square area of side M. If we denote K as the number of clusters, then on an average each cluster would have $\lfloor \frac{N}{K} \rfloor$ nodes. Let us also assume that each of the $\lfloor \frac{N}{K} \rfloor$ nodes in the cluster have L bits to transmit. For convenience, from hereon, we use $\frac{N}{K}$ instead of $\lfloor \frac{N}{K} \rfloor$.

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Further, if we assume $E_{cluster}$ as the energy consumption in a cluster, then

$$E_{cluster} = E_{local} + E_{LH} , \quad (1)$$

where E_{local} and E_{LH} denote the energy consumption for local and long haul transmission.

Considering the circuit and transmission energy costs separately, E_{Local} and E_{LH} can further be written as

$$\begin{aligned} E_{local} &= E_{c-local} + E_{Tx-local} \\ E_{LH} &= E_{c-LH} + E_{Tx-LH} , \end{aligned} \quad (2)$$

where $E_{c-local}$, $E_{Tx-local}$, E_{c-LH} and E_{Tx-LH} are the local circuit, local transmission, long haul circuit and long haul transmission energy costs.

Also, if we denote P_T, P_R as the power consumption in the transmitter and receiver, then

$$\begin{aligned} P_T &= P_{DAC} + P_{mix} + P_{filtx} \\ P_R &= P_{LNA} + P_{mix} + P_{IFA} + P_{filrx} + P_{ADC} \end{aligned} \quad (3)$$

where P_{DAC}, P_{mix}, P_{filtx}, P_{LNA}, P_{IFA}, P_{filrx} and P_{ADC} are the power consumption values in digital to analog converter (DAC), mixer circuit, transmitter filter, low noise amplifier (LNA), intermediate frequency amplifier (IFA), receiver filter and analog to digital converter (ADC).

To simplify our analysis, we neglect the power loss in sleep and transient modes.

3.1. Energy consumption for local information flow (E_{local}) within a cluster

In this subsection, we calculate the energy consumption for local information flow (E_{local}) per cluster. If we assume T_{on} as the ON time of the nodes [4] then

$$T_{on} \approx \frac{L}{Bb} ,$$

where b denotes the constellation size and B denotes modulation bandwidth.

Thus the circuit energy consumption (E_{C1}) per bit for a single node can be written as

$$\begin{aligned} E_{C1} &= \frac{P_{C1}T_{on}}{L} , &(4) \\ &= \frac{P_{C1}}{bB} , &(5) \end{aligned}$$

where

$$P_{C1} = P_T + \left(\frac{N}{K} - 1\right) P_R \quad (6)$$

is the circuit power consumption for local information flow. The factor of $\left(\frac{N}{K} - 1\right)$ in (6) accounts for the fact that in the cooperative MIMO transmission scheme, during the local information flow, there are always $\frac{N}{K} - 1$ nodes listening.

Since there are $\frac{N}{K}$ nodes on an average in a cluster, thus the total circuit energy per bit for local information flow can be written as

$$E_{c-local} = \frac{NP_{C1}}{KBb} . \quad (7)$$

Next, we consider the transmission energy required for the local information flow. For analysis throughout this paper, we assume the M-*ary* quadrature amplitude modulation (MQAM) as the underlying modulation scheme. Further, since the local distances are usually small, we assume the channel to be of additive white Gaussian noise (AWGN) type with square law path loss.

Using the upper bound on the probability of error for MQAM in [5], and using $\ln\left[\frac{4\left(1-2^{-b/2}\right)}{bP_b}\right] \leq \ln\left(\frac{2}{P_b}\right)$ for $b \geq 2$, we can upper bound the transmission energy per bit, for local information flow in an AWGN channel as

$$\begin{aligned} E_{Tx-local} \leq & \frac{2}{3}(1+\alpha)\left(\frac{2^b - 1}{b}\right)\ln\left(\frac{2}{P_b}\right) \times \\ & G_1 M_l N_f N_0 \sum_{i=1}^{\frac{N}{K}} \sum_{j=1,\, j\neq i}^{\frac{N}{K}} d_{ij}^2 , \end{aligned} \quad (8)$$

where $1 + \alpha = \frac{3}{\eta}\left[\frac{2^{\frac{b}{2}}-1}{2^{\frac{b}{2}}+1}\right]$ is the gain of the power amplifier, η is the drain efficiency of the power amplifier, P_b is the desired probability of error, G_1 is the antenna gain, M_l is the link margin, N_f is the noise figure, N_0 is the power spectral density of AWGN and d_{ij} is the mean distance between the nodes i and j respectively.

In general, the cluster could be of any arbitrary shape and hence the average distance between a node located at (x, y) and the center of mass of the cluster can be written as

$$\mathbb{E}\left[d_{CM}^2\right] = \int\int_{\mathcal{R}} \left(x^2 + y^2\right)\rho(x,y)\,dxdy , \quad (9)$$

where $\rho(x,y)$ denotes the node distribution at an arbitrary point (x, y) and $\mathbb{E}$ denotes the expectation operator.

Approximating the shape of the cluster as an ellipse, (with length of the major axis equal to c times the length of minor axis) and assuming uniform distribution of nodes with in the cluster i.e. $\left(\rho(x,y) = 1/\left(M^2/K\right)\right)$, we can show

$$\mathbb{E}\left[d_{CM}^2\right] = \frac{M^2\left(c^2+1\right)}{4\pi Kc} . \quad (10)$$

Since in an ellipse no two points are separated by a distance greater than the length of the major axis, hence we obtain the maximum distance between any two points in the cluster to be

$$\begin{aligned} d_{max}^2 &= 4 \times \mathbb{E}\left[d_{CM}^2\right] \\ &= \frac{M^2\left(c^2+1\right)}{\pi Kc} . \end{aligned} \quad (11)$$

We re-write (8) by replacing d_{ij}^2 with d_{max}^2 (of course this would make the bound in (8) loose), thus we obtain

$$E_{Tx-local} \leq \frac{N}{K^2}\left(\frac{N}{K} - 1\right)\frac{\left(2^{\frac{b}{2}} - 1\right)^2}{b}\epsilon_{local} , \quad (12)$$

where we define $\epsilon_{local} = \left(\frac{c^2+1}{\pi c}\right)\ln\left(\frac{2}{P_b}\right)\frac{G_1 M_l N_f N_0}{\eta} M^2$.

Thus, the total energy required to transmit L bits locally after approximating $E_{Tx-local}$ as an equality is

$$E_{local} = \frac{N}{K}\frac{P_{C1}}{Bb}L + \frac{N}{K^2}\left(\frac{N}{K} - 1\right)\frac{\left(2^{\frac{b}{2}} - 1\right)^2}{b}\epsilon_{local}L . \quad (13)$$

3.2. Energy consumption in the long haul (E_{LH}) per Cluster

In this subsection, we calculate the energy consumption in the long haul (E_{LH}) per cluster.

As in the case of energy analysis for local information flow, we can write the energy for long haul transmission (E_{LH}) as

$$E_{LH} = E_{C-LH} + E_{Tx-LH} , \quad (14)$$

where E_{C-LH} denotes the circuit energy consumption for long haul and E_{Tx-LH} denotes the transmission energy consumption for long haul.

The per bit circuit energy consumption for long haul transmission can be written as

$$E_{C-LH} = \frac{P_{C2} T_{on}}{L} , \quad (15)$$

where

$$P_{C2} = \frac{N}{K} P_T + P_R , \quad (16)$$

the factor of $\frac{N}{K}$ in (16) accounts for the cooperative transmission of $\frac{N}{K}$ nodes present in each cluster.

Since the long haul distances are large, hence we assume a fading channel with square law path loss. Using the upper bound for the probability of error for MQAM in Rayleigh fading in [5], we can upper bound E_{Tx-LH} per bit[1] as

$$E_{Tx-LH} \leq \frac{N}{K} \frac{\left(2^{\frac{b}{2}} - 1\right)^2}{b^{\frac{K}{N}+1}} \left(\frac{4}{P_b}\right)^{\frac{K}{N}} \epsilon_{LH} D^2 , \quad (17)$$

where we define

$$\epsilon_{LH} = \frac{2}{\eta} \frac{N_0 (4\pi)^2 M_l N_f}{G_t G_r \lambda^2} , \quad (18)$$

wherein G_t is the transmitter antenna gain, G_r is the receiver antenna gain and D is the long haul distance, it is assumed to be the same for all the nodes, since D is very large compared to the distance between individual nodes.

Thus, the total energy required to transmit $\frac{NL}{K}$ bits (since each node has L bits and there are $\frac{N}{K}$ nodes in each cluster) in the long haul after approximating the upper bound in (17) as an equality is

$$E_{LH} = \left(\frac{N}{K}\right)^2 L \frac{\left(2^{\frac{b}{2}} - 1\right)^2}{b^{\frac{K}{N}+1}} \left(\frac{4}{P_b}\right)^{\frac{K}{N}} \epsilon_{LH} D^2 + \left(\frac{P_{C2}}{bB}\right) \frac{NL}{K} . \quad (19)$$

3.3. Total Energy consumption (E_{total}) for cluster based cooperative MIMO transmission scheme

In this subsection, we calculate the total energy required for the cluster based cooperative MIMO scheme.

[1] In deriving this, we assumed a STBC with spatial rate equal to 1, so that there is no bandwidth expansion.

Using (1), (13) and (19), the maximum total energy consumption (E_{total}) for the cooperative MIMO scheme is

$$E_{total} = K E_{cluster}$$
$$= \frac{NL}{bB} \left[\left(\frac{N}{K} + 1\right) P_T + \left(\frac{N}{K}\right) P_R \right] + \frac{NL}{K} \frac{\left(2^{\frac{b}{2}} - 1\right)^2}{b} \times \left[\left(\frac{N}{K} - 1\right) M^2 \epsilon_{local} + N \left(\frac{4}{bP_b}\right)^{\frac{K}{N}} \epsilon_{LH} D^2 \right] . \quad (20)$$

Note that P_{C1} and P_{C2} are functions of the number of clusters K. It needs to be noted that the terms P_T and P_R in P_{C1} and P_{C2} are constants for a particular choice of transceiver.

4. OPTIMIZATION OF NUMBER OF CLUSTERS AND RATE

In this section, we present numerical techniques to determine the optimal number of clusters (K^*) and the optimal joint rate and number of clusters.

4.1. Numerical method to determine the optimal number of clusters

Here, we assume b is fixed and $b \geq 2$. We formulate the problem to determine the optimal number of clusters as follows:

$$\begin{array}{ll} \text{minimize} & E_{total} \\ \text{Subject to:} & 1 \leq K \leq N . \end{array} \quad (21)$$

It is straightforward to verify that $\frac{\partial^2 E_{total}}{\partial K^2} > 0$ for any $K > 0$ (see [6] for details), also the constraints in (21) are linear, thus the problem in (21) is a constrained convex optimization problem and can be efficiently solved by using the barrier method of the more general interior point methods [7]. After we obtain K^*, the optimal *integer* value is determined by evaluating E_{total} at the two neighboring integer points of K^*, and choosing the one that minimizes E_{total}.

4.2. Variation of Optimal Number of Clusters (K^*) with long haul distance (D)

Using the concept of super modular functions and its associated properties [8], K^* can be proved to be a decreasing function of long haul distance D (see [6] for details). Further as $D \to \infty$, the limiting value of K^* can be shown to be equal to $\frac{N}{\log\left(\frac{4}{bP_b}\right)}$ [6].

Since each cluster contains $\frac{N}{K}$ number of nodes, the decrease in K^* with respect to D implies an increase in number of nodes per cluster, thereby an increase in cooperation among the nodes to counter the path loss due to fading in the channel and maintain the desired probability of error. In general, as the performance requirement in terms of probability of error becomes more stringent (i.e. as the value of P_b decreases), the optimal number of clusters decreases, thus leading to an increase in *diversity* with in the cluster. This is further proved by our numerical results in Figure 1, where in we show the plot of K^* versus D, for a given rate ($b = 2$) and different values of probability of error (P_b).

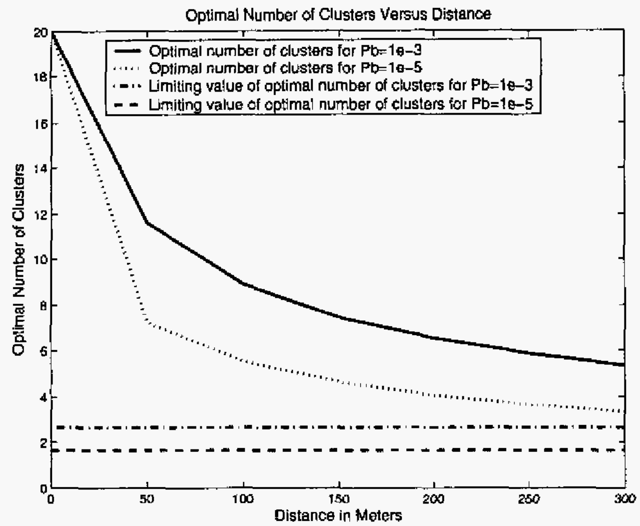

Fig. 1. The figure shows the plot of optimal number of clusters (K^*) versus long haul distance (D) for different values of probability of error (P_b) and a given rate (b).

4.3. Joint Rate and Cluster Optimization

We formulate the problem to determine the optimal joint rate and number of clusters ($[b\ K]^*$) as follows:

$$\begin{aligned} \underset{[b\ K]}{\text{minimize}} \quad & E_{total} \\ \text{Subject to:} \quad & 1 \leq K \leq N \\ & 2 \leq b \leq b_{max}\,, \end{aligned} \tag{22}$$

where b_{max} is the maximum rate at which a node can transmit and is determined by the maximum battery power of the node in the long haul. Using the calculations in [6] b_{max} was found to be 12 and is assumed in all of our numerical results.

In general, proving the convexity of $E_{total}\,(b, K)$ is difficult (since it requires proving the Hessian as positive semi definite).

We use the following algorithm to determine $[b\ K]^*$, whose convergence is proved subsequently. In solving (22) by Algorithm 1, we *relax* the assumption that $[b\ K]^*$ is integer valued.

Algorithm 1

For iterations $i = 1, 2, \cdots$ (until $E_{total}(i+1) - E_{total}(i) < \epsilon$) (for some tolerance $\epsilon > 0$)

1. Compute the current iterate value, $b^{i+1} = \arg\min_{\zeta \in X_1} E_{total}\left(\zeta, K^i\right)$, subject to the constraints in (22). X_1 is the feasible set for b.

2. Compute the current iterate value, $K^{i+1} = \arg\min_{\zeta \in X_2} E_{total}\left(b^{i+1}, \zeta\right)$, subject to the constraints in (22). X_2 is the feasible set for K.

The steps 1 and 2 in the above algorithm are solved using the barrier method of the more general interior point methods [7]. The optimal integer $[b\ K]^*$ is determined by evaluating E_{total} at the four neighboring integer points of the optimal solution to the *relaxed* optimization problem in (22) and then choosing the one that minimizes E_{total}.

The following theorem proves the convergence of Algorithm 1

Theorem 1 *The Algorithm 1 converges to a local minimum of the cost function in (22).*

PROOF. It can be proved that $\frac{\partial^2 E_{total}}{\partial K^2} \geq 0$ (see [6] for details), furthermore, it can be shown that $\frac{\partial^2 E_{total}}{\partial b^2} \geq 0$, for $b \geq 2$ (see [6] for details), hence by proposition 2.7.1 in [9], the Algorithm 1 converges to a local minimum of the cost function in (22). □

4.4. Numerical Results

The system parameter values used for our numerical results are same as in [1, 4]. To calculate the values for P_{ADC} and P_{DAC}, we used the formulas provided in the Appendix of [4]. We also assume $M = 10$ m, $c = 2$, (i.e. we assume the cluster as an ellipse, with the length of its major axis equal to twice the length of its minor axis) and $L = 10$ Kb. Also, for the cluster alone optimization scheme we set $b = 2$. For the rate only optimized scheme number of clusters (K) equal to one.

Figure 2 shows the plot of minimized E_{total} versus D, for N equal to 4 and 20 respectively, for rate only and cluster only optimized schemes. For $N = 4$, the cluster only and rate only optimized schemes consume almost the same amount of energy in the long haul. For $N = 20$, the cluster based cooperative MIMO scheme is more energy efficient compared to the rate only optimized scheme. Thus, in a large scale sensor network employing a cluster based cooperative MIMO scheme can lead to large scale energy savings compared to a rate optimized scheme.

Figure 3 shows the plot of minimized E_{total} versus D, for $N = 20$, for a rate only, cluster only and joint rate and cluster optimized schemes. The results show the efficacy of a joint rate and cluster optimized scheme over a rate only or cluster only optimized schemes (especially for short and medium range applications). But in long haul, the energy consumption of cluster only and joint rate and cluster optimized schemes are almost equal.

5. IMPLEMENTATION ISSUES

In this section, we discuss the implementation issues of the proposed cluster based cooperative MIMO scheme for the sensor networks.

In [3], it was observed that constructing the orthogonal STBC for complex modulation schemes (such as PSK, QAM) with a *spatial rate of one* is not possible when number of antennas are greater than two. But even for such cases, it is possible to construct a STBC that would result in a *spatial rate of* $1/2$. Also, in recent years, there has been some work on constructing complex STBC for certain sporadic number of antennas, that would result in a *spatial rate greater than* $1/2$. For example, in [10, 11] the author presents a complex orthogonal STBC for 7 and 8 transmit antennas, with a *spatial rate equal to* $5/8$. Of course, using such *reduced spatial rate* STBC can lead to an increase in the bandwidth of the system.

However as mentioned in [3], it is still possible to construct orthogonal STBC for *real* modulation schemes such as pulse amplitude modulation (PAM) with a *spatial rate equal to one*. But,

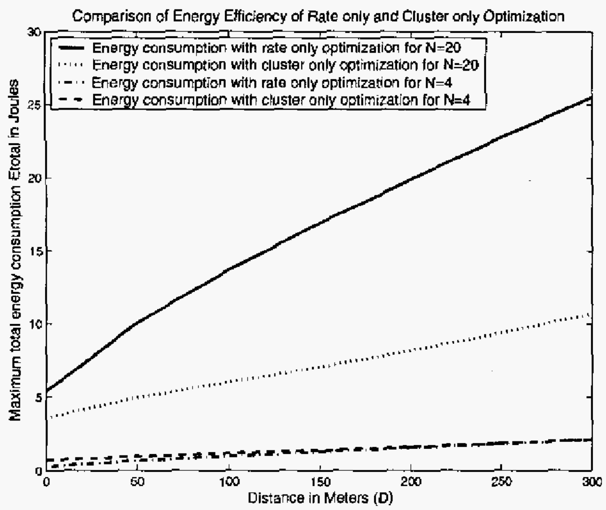

Fig. 2. The figure shows the comparison of optimized total energy consumption versus long haul distance for rate only and cluster only optimized transmission Schemes for $N = 4$ and $N = 20$ respectively.

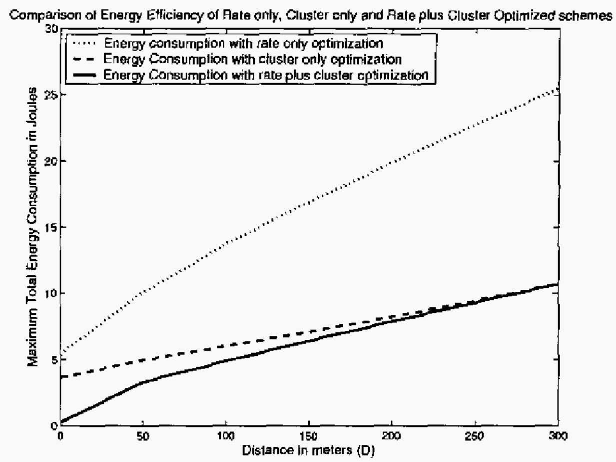

Fig. 3. The figure shows the comparison of optimized total energy consumption versus long haul distance for rate only, cluster only and joint rate and cluster optimized transmission Schemes for $N = 20$.

PAM is inherently power inefficient and hence is not suitable for energy limited sensor networks.

6. CONCLUSIONS

We considered energy efficiency analysis of cluster based cooperative MIMO sensor networks. Using an expression for total energy consumption in sensor networks it was found that, as the long haul distance increases the optimal number of clusters decreases, thereby proving that as the long haul distance increases cooperation among the nodes increases. Further, the limiting value of optimal number of clusters was shown to converge to a constant. Next, we propose a provably convergent block coordinate descent algorithm to determine the optimal joint rate and number of clusters. Through our numerical results, we observe that in a large scale sensor network, the cluster only optimized scheme can be more energy efficient than a rate only optimized scheme. Further, it had been observed that employing a joint rate and cluster optimized scheme can lead to large scale energy savings for short and medium range applications, and in the long haul, its performance had been found to be similar to a cluster only optimized scheme.

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